# Drinfeld basis of the twisted quantum affine algebra $U_q(A^{(2)}_2)$ from the Gauss decomposition of an $L$-operator



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**LETTER TO THE EDITOR**

# Drinfeld basis of the twisted quantum affine algebra $U_q(A_2^{(2)})$ from the Gauss decomposition of an $L$-operator

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**Abstract**

We establish the algebraic isomorphism between the Drinfeld basis of the twisted quantum affine algebra $U_q(A_2^{(2)})$ and the twisted Reshetihkin and Semenov-Tian-Shansky algebra by using the Gauss decomposition technique of Ding and Frenkel.

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## 1. Introduction

For quantum affine (super) algebras, there exist two types of loop realization: Reshetihkin and Semenov-Tian-Shansky (RS) realization (or the RS algebras) [1, 2] and the Drinfeld bases [3] of quantum affine (super) algebras. Actually, there exists an algebraic isomorphism between the Drinfeld base and the RS algebra realization of quantum affine (super) algebra [4]. The isomorphism between Drinfeld bases and the RS realization was established by the Gauss decomposition of the corresponding $L$-operators for $U_q(\widehat{gl(n)})$ [4], for $U_q[gl(m|n)^{(1)}]$ [2, 5], $U_q[osp(1|2)^{(1)}]$ [6] and $U_q[osp(2|2)^{(2)}]$ [7]. In this letter, we shall extend such an algebraic isomorphism to the twisted quantum affine algebra case. We establish the algebraic isomorphism between the Drinfeld basis of the twisted quantum affine algebra $U_q(A_2^{(2)})$ and the twisted RS algebra by using the Gauss decomposition technique of Ding and Frenkel.

## 2. The twisted quantum affine algebra $U_q(A_2^{(2)})$

The symmetric Cartan matrix of the twisted affine Lie algebra $A_2^{(2)}$ is

$$(a_{ij}) = \begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix}$$

where $i, j = 0, 1$. The quantum affine algebra $U_q(A_2^{(2)})$ is a $q$-analogue of the universal enveloping algebra of $A_2^{(2)}$ generated by the Chevalley generators $\{e_i, f_i, t_i^{\pm 1}, d | i = 0, 1\}$, where $d$ is the usual derivation operator. The defining relations are [8]

$$t_i t_j = t_j t_i \qquad t_i d = d t_i$$

$$[d, e_i] = \delta_{i,0} e_i \qquad [d, f_i] = -\delta_{i,0} f_i$$

$$t_i e_j t_i^{-1} = (q^{\frac{1}{2}})^{a_{ij}} e_j \qquad t_i f_j t_i^{-1} = (q^{\frac{1}{2}})^{-a_{ij}} f_j$$

$$[e_i, f_j] = \delta_{ij} \frac{t_i - t_i^{-1}}{q_i - q_i^{-1}}$$

$$\sum_{r=0}^{1-a_{ij}} (-1)^r \begin{bmatrix} 1 - a_{ij} \\ r \end{bmatrix}_{q_i} (e_i)^r e_j (e_i)^{1-a_{ij}-r} = 0 \qquad \text{if} \quad i \neq j$$

$$\sum_{r=0}^{1-a_{ij}} (-1)^r \begin{bmatrix} 1 - a_{ij} \\ r \end{bmatrix}_{q_i} (f_i)^r f_j (f_i)^{1-a_{ij}-r} = 0 \qquad \text{if} \quad i \neq j$$

where $q_1 = q^{\frac{1}{2}}$, $q_0 = q^2$, $t_i = q_i^{h_i}$, and

$$[n]_{q_i} = \frac{q_i^n - q_i^{-n}}{q_i - q_i^{-1}} \qquad [n]_{q_i}! = [n]_{q_i}[n-1]_{q_i} \cdots [1]_{q_i}$$
$$\begin{bmatrix} n \\ r \end{bmatrix}_{q_i} = \frac{[n]_{q_i}!}{[n-r]_{q_i}! [r]_{q_i}!}. \tag{2.1}$$

$U_q(A_2^{(2)})$ is a quasi-triangular Hopf algebra endowed with Hopf algebra structure:

$$\Delta(t_i) = t_i \otimes t_i \qquad \Delta(e_i) = e_i \otimes 1 + t_i \otimes e_i$$

$$\Delta(f_i) = f_i \otimes t_i^{-1} + 1 \otimes f_i$$

$$\epsilon(t_i) = 1 \qquad \epsilon(e_i) = \epsilon(f_i) = 0$$

$$S(e_i) = -t_i^{-1} e_i \qquad S(f_i) = -f_i t_i$$

$$S(t_i^{\pm 1}) = t_i^{\mp 1} \qquad S(d) = -d.$$

$U_q(A_2^{(2)})$ can also be realized by the Drinfeld generators [9] $\{X^{\pm}(z),\ \phi(z),\ \psi(z)\}$.

**Definition 1.** *$U_q(A_2^{(2)})$ is an associative algebra with unit 1 and the Drinfeld generators: $X^{\pm}(z)$, $\phi(z)$ and $\psi(z)$, and a central element $c$. $\phi(z)$ and $\psi(z)$ are invertible. The defining* *relations are given by*

$$
\begin{aligned}
&\phi(z)\phi(w)=\phi(w)\phi(z)\\
&\psi(z)\psi(w)=\psi(w)\psi(z)\\
&\phi(z)\psi(w)\phi(z)^{-1}\psi(w)^{-1}=\frac{(z_+q+w_-)(z_-+w_+q)(z_+-w_-q^2)(z_-q^2-w_+)}{(z_++w_-q)(z_-q+w_+)(z_+q^2-w_-)(z_--w_+q^2)}\\
&\phi(z)X^-(w)\phi(z)^{-1}=\frac{(z_+-wq^2)(z_+q+w)}{(z_+q^2-w)(z_++wq)}X^-(w)\\
&\phi(z)^{-1}X^+(w)\phi(z)=\frac{(z_--wq^2)(z_-q+w)}{(z_-q^2-w)(z_-+wq)}X^+(w)\\
&\psi(z)X^-(w)\psi(z)^{-1}=\frac{(z_--wq^2)(z_-q+w)}{(z_-q^2-w)(z_-+wq)}X^-(w)\\
&\psi(z)^{-1}X^+(w)\psi(z)=\frac{(z_+-wq^2)(z_+q+w)}{(z_+q^2-w)(z_++wq)}X^+(w)\\
&(z+wq)(zq^2-w)X^-(z)X^-(w)=(zq+w)(z-wq^2)X^-(w)X^-(z)\\
&(zq+w)(z-wq^2)X^+(z)X^+(w)=(z+wq)(zq^2-w)X^+(w)X^+(z)\\
&[X^+(z),X^-(w)]=\frac{1}{q-q^{-1}}\left[\delta\left(\frac{w}{z}q^c\right)\psi(w_+)-\delta\left(\frac{w}{z}q^{-c}\right)\phi(z_+)\right]
\end{aligned}
\tag{2.2}
$$

*where* $z_\pm = zq^{\pm c/2}$.

Here, and throughout,

$$\delta(z)=\sum_{l\in\mathbb{Z}} z^l \tag{2.3}$$

is a formal series. $\delta(z)$ enjoys the following properties:

$$\delta\left(\frac{z}{w}\right)=\delta\left(\frac{w}{z}\right) \qquad \delta\left(\frac{z}{w}\right)f(z)=\delta\left(\frac{z}{w}\right)f(w). \tag{2.4}$$

The corresponding Drinfeld currents $\psi^\pm(z)$ and $X^\pm(z)$ are defined by

$$
\begin{aligned}
\psi^+(z)&=\sum_{m=0}^{\infty}\psi_m^+z^{-m}=K\exp\left\{(q_1-q_1^{-1})\sum_{k=1}^{\infty}a_kz^{-k}\right\}\\
\psi^-(z)&=\sum_{m=0}^{\infty}\psi_{-m}^-z^{m}=K^{-1}\exp\left\{-(q_1-q_1^{-1})\sum_{k=1}^{\infty}a_{-k}z^{k}\right\}\\
X^\pm(z)&=\sum_{n\in Z}X_m^\pm z^{-m}.
\end{aligned}
$$

The Chevalley generators are related to the Drinfeld generators by the formulae:

$$
\begin{aligned}
&t_1=K \qquad e_1=X_0^+ \qquad t_0=\gamma K^{-2} \qquad f_1=X_0^-\\
&e_0=K^{-2}[X_0^-,X_1^-]_q \qquad f_0=\frac{1}{[4]_{q_1}^2}[X_{-1}^+,X_0^+]_{q^{-1}}K^2.
\end{aligned}
$$

## 3. Derivation of the Drinfeld basis from the RS algebra

Let us recall the definition of the RS algebra. Let $R(\frac{z}{w})\in \mathrm{End}(V\otimes V)$, where $V$ is a vector space, be a $R$-matrix which satisfies the Yang–Baxter equation

$$R_{12}(z)R_{13}(zw)R_{23}(w)=R_{23}(w)R_{13}(zw)R_{12}(z). \tag{3.1}$$

We introduce the following definition from [1].

**Definition 2.** *The RS algebra $U(\mathcal{R})$ is generated by an invertible L-operator $L^{\pm}(z)$, which obeys the relations*

$$R\left(\frac{z}{w}\right)L_1^{\pm}(z)L_2^{\pm}(w)=L_2^{\pm}(w)L_1^{\pm}(z)R\left(\frac{z}{w}\right) \tag{3.2}$$

$$R\left(\frac{z_+}{w_-}\right)L_1^{+}(z)L_2^{-}(w)=L_2^{-}(w)L_1^{+}(z)R\left(\frac{z_-}{w_+}\right) \tag{3.3}$$

*where $L_1^{\pm}(z)=L^{\pm}(z)\otimes 1$, $L_2^{\pm}(z)=1\otimes L^{\pm}(z)$. For (3.2), the expansion direction of $R(z/w)$ can be chosen in $z/w$ or in $w/z$, but for (3.3), the expansion direction must only be in $z/w$.*

For the level-one case, the RS algebra of $U_q(A_2^{(2)})$ was realized by Miki's construction of a level-one vertex operator of $U_q(A_2^{(2)})$ [10].

In the following we apply the RS algebra to derive the Drinfeld basis of $U_q(A_2^{(2)})$ by using the Gauss decomposition technique. We take $R(\frac{z}{w})$ to be the $R$-matrix associated to the three-dimensional vector representation of $U_q(A_2^{(2)})$. The $R$-matrix has the following form:

$$R\left(\frac{z}{w}\right)=\begin{pmatrix}1&0&0&0&0&0&0&0&0\\0&a&0&b&0&0&0&0&0\\0&0&d&0&c&0&r&0&0\\0&f&0&a&0&0&0&0&0\\0&0&g&0&e&0&c&0&0\\0&0&0&0&0&a&0&b&0\\0&0&s&0&g&0&d&0&0\\0&0&0&0&0&f&0&a&0\\0&0&0&0&0&0&0&0&1\end{pmatrix} \tag{3.4}$$

where

$$\begin{aligned}
&a=\frac{q(z-w)}{zq^2-w}\qquad b=\frac{w(q^2-1)}{zq^2-w}\qquad c=\frac{q^{5/2}z(q^2-1)(z-w)}{(zq^2-w)(zq^3+w)}\\
&d=\frac{q^2(z-w)(zq+w)}{(zq^2-w)(zq^3+w)}\qquad e=a-\frac{zw(q^2-1)(q^3+1)}{(zq^2-w)(zq^3+w)}\\
&f=\frac{w(q^2-1)}{zq^2-w}\qquad g=\frac{q^{1/2}w(q^2-1)(z-w)}{(zq^2-w)(zq^3+w)}\\
&r=\frac{z(1-q^2)[q^3z-q^2(z-w)+w]}{(zq^2-w)(zq^3+w)}\\
&s=\frac{w(1-q^2)[q^3z+q(z-w)+w]}{(zq^2-w)(zq^3+w)}.
\end{aligned} \tag{3.5}$$

We use the following decomposition for $L^{\pm}(z)$:

$$\begin{aligned}
L^{\pm}(z)&=\begin{pmatrix}1&0&0\\e_1^{\pm}(z)&1&0\\e_{3,1}^{\pm}(z)&e_2^{\pm}(z)&1\end{pmatrix}\begin{pmatrix}k_1^{\pm}(z)&0&0\\0&k_2^{\pm}(z)&0\\0&0&k_3^{\pm}(z)\end{pmatrix}\begin{pmatrix}1&f_1^{\pm}(z)&f_{1,3}^{\pm}(z)\\0&1&f_2^{\pm}(z)\\0&0&1\end{pmatrix}\\
&=\begin{pmatrix}k_1^{\pm}(z)&k_1^{\pm}(z)f_1^{\pm}(z)&k_1^{\pm}(z)f_{1,3}^{\pm}(z)\\e_1^{\pm}(z)k_1^{\pm}(z)&k_2^{\pm}(z)+e_1^{\pm}(z)k_1^{\pm}(z)f_1^{\pm}(z)&u^{\pm}\\e_{3,1}^{\pm}(z)k_1^{\pm}(z)&v^{\pm}&x^{\pm}\end{pmatrix}
\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}
u^{\pm}&=k_2^{\pm}(z)f_2^{\pm}(z)+e_1^{\pm}(z)k_1^{\pm}(z)f_{1,3}^{\pm}(z)\\
v^{\pm}&=e_2^{\pm}(z)k_2^{\pm}(z)+e_{3,1}^{\pm}(z)k_1^{\pm}(z)f_1^{\pm}(z)\\
x^{\pm}&=k_3^{\pm}(z)+e_2^{\pm}(z)k_2^{\pm}(z)f_2^{\pm}(z)+e_{3,1}^{\pm}(z)k_1^{\pm}(z)f_{1,3}^{\pm}(z).
\end{aligned} \tag{3.7}$$

Let us define the total currents

$$X_i^+(z) = f_i^+(z_+) - f_i^-(z_-) \qquad i = 1, 2 \tag{3.8}$$

$$X_i^-(z) = e_i^-(z_+) - e_i^+(z_-) \qquad i = 1, 2. \tag{3.9}$$

The inversions $L^{\pm}(z)^{-1}$ of (3.6) are easily seen to be

$$L^{\pm}(z)^{-1} = \begin{pmatrix} y^{\pm} & \tilde{x}^{\pm} & \tilde{u}^{\pm} \\ \tilde{y}^{\pm} & k_2^{\pm}(z)^{-1} + f_2^{\pm}(z)k_3^{\pm}(z)^{-1}e_2^{\pm}(z) & -f_2^{\pm}(z)k_3^{\pm}(z)^{-1} \\ \tilde{v}^{\pm} & -k_3^{\pm}(z)^{-1}e_2^{\pm}(z) & k_3^{\pm}(z)^{-1} \end{pmatrix} \tag{3.10}$$

where

$$\begin{aligned} \tilde{u}^{\pm} &= [f_1^{\pm}(z)f_2^{\pm}(z) - f_{1,3}^{\pm}(z)]k_3^{\pm}(z)^{-1} \\ \tilde{v}^{\pm} &= k_3^{\pm}(z)^{-1}[e_2^{\pm}(z)e_1^{\pm}(z) - e_{3,1}^{\pm}(z)] \\ y^{\pm} &= k_1^{\pm}(z)^{-1} + f_1^{\pm}(z)k_2^{\pm}(z)^{-1}e_1^{\pm}(z) + [f_1^{\pm}(z)f_2^{\pm}(z) - f_{1,3}^{\pm}(z)] \\ &\qquad \times k_3^{\pm}(z)^{-1}[e_2^{\pm}(z)e_1^{\pm}(z) - e_{3,1}^{\pm}(z)] \\ \tilde{x}^{\pm} &= -f_1^{\pm}(z)k_2^{\pm}(z)^{-1} + [f_{1,3}^{\pm}(z) - f_1^{\pm}(z)f_2^{\pm}(z)]k_3^{\pm}(z)^{-1}e_2^{\pm}(z) \\ \tilde{y}^{\pm} &= -k_2^{\pm}(z)^{-1}e_1^{\pm}(z) + f_2^{\pm}(z)k_3^{\pm}(z)^{-1}[e_{3,1}^{\pm}(z) - e_2^{\pm}(z)e_1^{\pm}(z)]. \end{aligned} \tag{3.11}$$

By the definition of the RS algebra and the Gauss decomposition formula (3.6), and after tedious calculations, we derive

$$\begin{aligned} &k_1^{\pm}(z)k_1^{\pm}(w) = k_1^{\pm}(w)k_1^{\pm}(z) \\ &k_1^{+}(z)k_1^{-}(w) = k_1^{-}(w)k_1^{+}(z) \\ &k_2^{\pm}(z)k_2^{\pm}(w) = k_2^{\pm}(w)k_2^{\pm}(z) \\ &k_3^{\pm}(z)k_3^{\pm}(w) = k_3^{\pm}(w)k_3^{\pm}(z) \\ &k_3^{+}(z)k_3^{-}(w) = k_3^{-}(w)k_3^{+}(z) \\ &k_1^{\pm}(z)k_2^{\pm}(w) = k_2^{\pm}(w)k_1^{\pm}(z) \\ &\frac{z_{\pm} - w_{\mp}}{z_{\pm}q^2 - w_{\mp}}k_1^{\pm}(z)k_2^{\mp}(w) = \frac{z_{\mp} - w_{\pm}}{z_{\mp}q^2 - w_{\pm}}k_2^{\mp}(w)k_1^{\pm}(z) \\ &k_1^{\pm}(z)k_3^{\pm}(w)^{-1} = k_3^{\pm}(w)^{-1}k_1^{\pm}(z) \\ &\frac{(z_{\mp} - w_{\pm})(z_{\mp}q + w_{\pm})}{(z_{\mp}q^2 - w_{\pm})(z_{\mp}q^3 + w_{\pm})}k_1^{\pm}(z)k_3^{\mp}(w)^{-1} = \frac{(z_{\pm} - w_{\mp})(z_{\pm}q + w_{\mp})}{(z_{\pm}q^2 - w_{\mp})(z_{\pm}q^3 + w_{\mp})}k_3^{\mp}(w)^{-1}k_1^{\pm}(z) \\ &\frac{z_{\pm} + w_{\mp}q}{z_{\pm}q + w_{\mp}}k_2^{\pm}(z)k_2^{\mp}(w) = \frac{z_{\mp} + w_{\pm}q}{z_{\mp}q + w_{\pm}}k_2^{\mp}(w)k_2^{\pm}(z) \\ &k_2^{\pm}(z)^{-1}k_3^{\pm}(w)^{-1} = k_3^{\pm}(w)^{-1}k_2^{\pm}(z)^{-1} \\ &\frac{z_{\pm} - w_{\mp}}{z_{\pm}q^2 - w_{\mp}}k_2^{\pm}(z)^{-1}k_3^{\mp}(w)^{-1} = \frac{z_{\mp} - w_{\pm}}{z_{\mp}q^2 - w_{\pm}}k_3^{\mp}(w)^{-1}k_2^{\pm}(z)^{-1} \end{aligned} \tag{3.12}$$

$$
\begin{aligned}
k_1^{\pm}(z)X_1^-(w)k_1^{\pm}(z)^{-1} &= \frac{z_{\pm}q^2 - w}{q(z_{\pm} - w)}X_1^-(w)\\
k_1^{\pm}(z)^{-1}X_1^+(w)k_1^{\pm}(z) &= \frac{z_{\mp}q^2 - w}{q(z_{\mp} - w)}X_1^+(w)\\
k_2^{\pm}(z)X_1^-(w)k_2^{\pm}(z)^{-1} &= \frac{(z_{\pm} - wq^2)(z_{\pm}q + w)}{q(z_{\pm} - w)(z_{\pm} + wq)}X_1^-(w)\\
k_2^{\pm}(z)^{-1}X_1^+(w)k_2^{\pm}(z) &= \frac{(z_{\mp} - wq^2)(z_{\mp}q + w)}{q(z_{\mp} - w)(z_{\mp} + wq)}X_1^+(w)\\
k_3^{\pm}(z)X_1^-(w)k_3^{\pm}(z)^{-1} &= \frac{z_{\pm} + wq^3}{q(z_{\pm} + wq)}X_1^-(w)\\
k_3^{\pm}(z)^{-1}X_1^+(w)k_3^{\pm}(z) &= \frac{z_{\mp} + wq^3}{q(z_{\mp} + wq)}X_1^+(w)\\
k_1^{\pm}(z)X_2^-(w)k_1^{\pm}(z)^{-1} &= \frac{z_{\pm}q^3 + w}{q(z_{\pm}q + w)}X_2^-(w)\\
k_1^{\pm}(z)^{-1}X_2^+(w)k_1^{\pm}(z) &= \frac{z_{\mp}q^3 + w}{q(z_{\mp}q + w)}X_2^+(w)\\
k_2^{\pm}(z)X_2^-(w)k_2^{\pm}(z)^{-1} &= \frac{(z_{\pm} + wq)(z_{\pm}q^2 - w)}{q(z_{\pm}q + w)(z_{\pm} - w)}X_2^-(w)\\
k_2^{\pm}(z)^{-1}X_2^+(w)k_2^{\pm}(z) &= \frac{(z_{\mp} + wq)(z_{\mp}q^2 - w)}{q(z_{\mp}q + w)(z_{\mp} - w)}X_2^+(w)\\
k_3^{\pm}(z)X_2^-(w)k_3^{\pm}(z)^{-1} &= \frac{z_{\pm} - wq^2}{q(z_{\pm} - w)}X_2^-(w)\\
k_3^{\pm}(z)^{-1}X_2^+(w)k_3^{\pm}(z) &= \frac{z_{\mp} - wq^2}{q(z_{\mp} - w)}X_2^+(w)
\end{aligned}
\tag{3.13}
$$

$$
\begin{aligned}
&(z - w)(zq^3 + w)X_1^-(z)X_2^-(w) = (z + wq)(zq^2 - w)X_2^-(w)X_1^-(z)\\
&(z + wq)(zq^2 - w)X_1^-(z)X_1^-(w) = (zq + w)(z - wq^2)X_1^-(w)X_1^-(z)\\
&(z + wq)(zq^2 - w)X_2^-(z)X_2^-(w) = (zq + w)(z - wq^2)X_2^-(w)X_2^-(z)\\
&(z + wq)(zq^2 - w)X_1^+(z)X_2^+(w) = (z - w)(zq^3 + w)X_2^+(w)X_1^+(z)\\
&(z - wq^2)(zq + w)X_1^+(z)X_1^+(w) = (z + wq)(zq^2 - w)X_1^+(w)X_1^+(z)\\
&(z - wq^2)(zq + w)X_2^+(z)X_2^+(w) = (z + wq)(zq^2 - w)X_2^+(w)X_2^+(z)
\end{aligned}
\tag{3.14}
$$

$$
\begin{aligned}
&[X_1^+(z), X_1^-(w)] = (q - q^{-1})\left[-\delta\left(\frac{z}{w}q^c\right)k_2^+(z_+)k_1^+(z_+)^{-1} + \delta\left(\frac{z}{w}q^{-c}\right)k_2^-(w_+)k_1^-(w_+)^{-1}\right]\\
&[X_2^+(z), X_2^-(w)] = (q - q^{-1})\left[\delta\left(\frac{z}{w}q^c\right)k_3^+(z_+)k_2^+(z_+)^{-1} - \delta\left(\frac{z}{w}q^{-c}\right)k_3^-(w_+)k_2^-(w_+)^{-1}\right]\\
&[X_1^+(z), X_2^-(w)] = (q - q^{-1})q^{\frac{1}{2}}\left[-\delta\left(-\frac{z}{w}q^{c+1}\right)k_2^+(z_+)k_1^+(z_+)^{-1}\right.\\
&\qquad\left. + \delta\left(-\frac{z}{w}q^{-c+1}\right)k_3^-(w_+)k_2^-(w_+)^{-1}\right]\\
&[X_2^+(z), X_1^-(w)] = (q - q^{-1})q^{-\frac{1}{2}}\left[\delta\left(-\frac{z}{w}q^{c-1}\right)k_2^+(-z_+q^{-1})k_1^+(-z_+q^{-1})^{-1}\right.\\
&\qquad\left. - \delta\left(-\frac{z}{w}q^{-c-1}\right)k_3^-(-w_+q)k_2^-(-w_+q)^{-1}\right].
\end{aligned}
\tag{3.15}
$$

We define the algebraic isomorphism

$$
\begin{aligned}
X^{\pm}(z) &= (q - q^{-1})\left[X_1^{\pm}(z) + X_2^{\pm}(-zq)\right] \\
\phi(z) &= (1 + q^{\frac{1}{2}} - q^{-\frac{1}{2}})\phi_1(z) - \phi_2(-zq) \\
\psi(z) &= \psi_1(z) - (1 + q^{-\frac{1}{2}} - q^{\frac{1}{2}})\psi_2(-zq)
\end{aligned}
\tag{3.16}
$$

where

$$
\begin{aligned}
\phi_i(z) &= k_{i+1}^+(z)k_i^+(z)^{-1} \\
\psi_i(z) &= k_{i+1}^-(z)k_i^-(z)^{-1} \qquad i = 1, 2.
\end{aligned}
\tag{3.17}
$$

Then $X^{\pm}(z)$, $\phi(z)$, $\psi(z)$ satisfy the Drinfeld current commutation relations of $U_q(A_2^{(2)})$. This completes the derivation of the Drinfeld basis from the RS algebra in the $U_q(A_2^{(2)})$ case.

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